ERG2011A (2006 Fall)

Tutorial 1

Ordinary Differential Equation

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Course Information:

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1. Advanced Calculus Review

i. $$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

ii. $$\int \sin x\, dx = -\cos x + C$$

iii. $$\int \cos x\, dx = \sin x + C$$

iv. $$\int \tan x\, dx = -\ln|\cos x| + C$$

v. $$\int \sec x\, dx = \ln|\sec x + \tan x| + C$$

vi. $$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

vii. $$\int \frac{1}{x} dx = \ln|x| + C$$

viii. $$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

2. Introduction of Differential Equation

Differential Equation: Equation of involve derivatives, and we want to solve for some unknown function y(x) from the equation.

Examples:

$$y' + p(x)y + y''' = q(x) \text{---------} (*)$$

Below are some definitions and properties of a differential equation:

A. Ordinary Differential Equation & Partial Differential Equation

Differential equation that involves ONLY ONE independent variables is called Ordinary, just like equation (*). Otherwise, it is called Particular.

B. Order, Degree And Linearity

Order: order of the highest derivatives in the differential equation.

Degree: power of the highest derivative terms.

Linear: No multiplications among dependent variables/derivatives. All coefficients are functions of independent variables.

So (*) is differential equation of order 3, degree 1 and linear.

C. Homogeneous And Non-homogeneous

For the differential equation which R.H.S. is equal to 0, it is called homogeneous. So (*) is homogeneous equation if q(x) = 0, otherwise, it is non-homogeneous.

D. General Solution, Particular Solution

For example, for y'=x, its general solution is in the format

$$y(x) = \frac{x^2}{2} + C$$

C can be any real number, which gives out different members from that family of curves.

Particular solution is the solution derived from general solution which the constant has specific values, e.g.

$$y(x) = \frac{x^2}{2} + 6$$

Usually, the ***Initial Condition*** or ***Boundary Condition*** is given to find out the value of the constants.

3. Methods Of Solving Differential Equations

A. Graphical Method

--Plot out the direction field line-by-line and estimate the graph. (See lecture notes)

--Slow, not efficient, but can estimate solutions of complex format.

B. Separable Form

y’ = f(x, y) , f(x,y) = g(x) * h(y)

$$\int g(x)dx = \int \frac{1}{h(y)} dy$$

Exercises: y’ = (x+3)(y+1)

C. Substitution To Separable

--Use some substitution to reduce the differential equation into separable form.

Common examples are $u = \frac{y}{x}$, $u = ax + by + c$

Exercises: 1) 2 xyy' = (y-x)(y+x)   (question in lecture slides)

2) $y' = \frac{1 - 2y - 4x}{1 + y + 2x}$ (Set 1.3, Q25)

D. Exact Equation

P(x,y) dx + Q(x,y) dy = 0

--Text "Exactness": $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$?

--Then solve u(x,y) from $\frac{du}{dx} = P$

-- u(x,y) = C is a general solution.

Exercises: $-yx^{-2}dx + x^{-1}dy = 0$ (Set 1.5, Q8)

E. Finding Integrating Factor

--Find some integrating factor, multiply to the whole equation to make it become exact.

$$\frac{1}{F_1(x)}\frac{dF_1(x)}{dx} = \frac{1}{Q(x,y)}\left(\frac{\partial P(x,y)}{\partial y} - \frac{\partial Q(x,y)}{\partial x}\right)$$

OR

$$\frac{1}{F_2(y)}\frac{dF_2(y)}{dy} = \frac{1}{P(x,y)}\left(\frac{\partial Q(x,y)}{\partial x} - \frac{\partial P(x,y)}{\partial y}\right)$$

Then $F_1(x)$ or $F_2(y)$ is called the integrating factor. You can also try to let the integrating factor in the format $x^m y^n$, and solve for m and n.

Examples: $2x \tan y dx + \sec^2 y dy + 0$ (Set 1.5, Q35)