$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\therefore$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

Let $\tan\theta = c_0 + c_1\theta + c_2\theta^2 + c_3\theta^3 + \dots$

$\because \tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta \cdot \cos\theta = \sin\theta$

$$\Rightarrow (c_0 + c_1\theta + c_2\theta^2 + c_3\theta^3 + \dots)\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots\right)$$

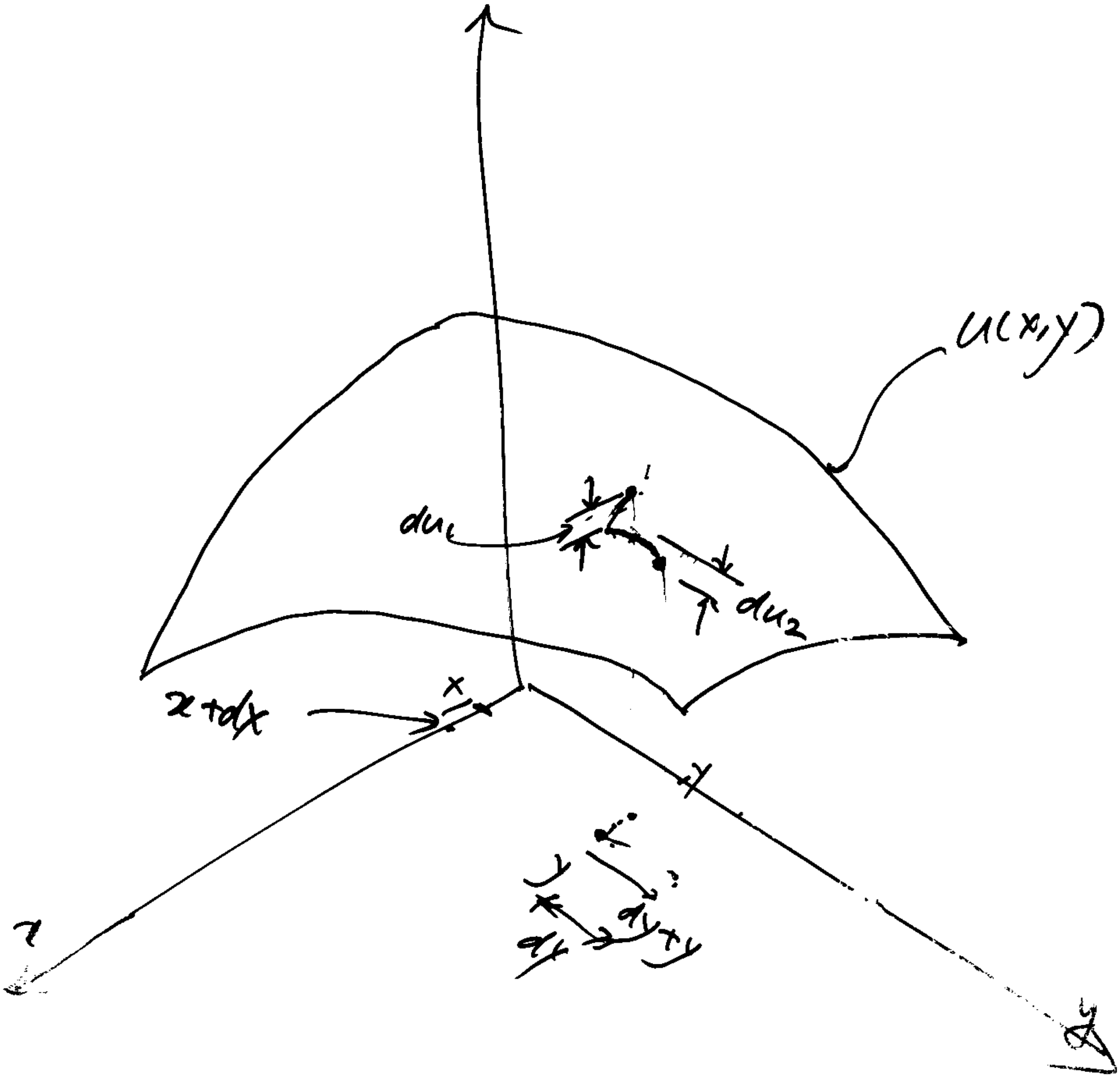
$$= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

By Comparing Coefficients, we have

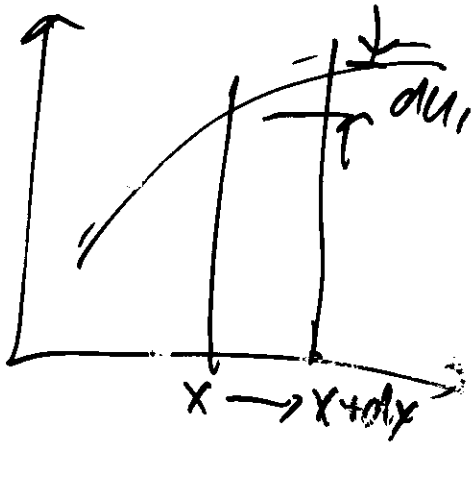
$c_0 = 0$, $c_1 = 1$, $c_2 = 0$

$$c_3 - c_1 \cdot \frac{1}{2!} = -\frac{1}{3!}$$

$$c_3 = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}, \dots$$



$$du = du_1 + du_2$$



$$du_1 = \frac{du_1}{dx} \cdot dx$$

$$\boxed{\frac{\partial u}{\partial x} dx}$$

$$\frac{\partial u}{\partial y} \cdot dy$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \dots$$

$$= \left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots\right) + i\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \dots\right)$$

$$e^{i\theta} \triangleq \cos\theta + i\sin\theta$$

$$y = \sin x$$

$$\boxed{yx^3 + x^2y + y^2 = \text{const}}$$

is a solution to

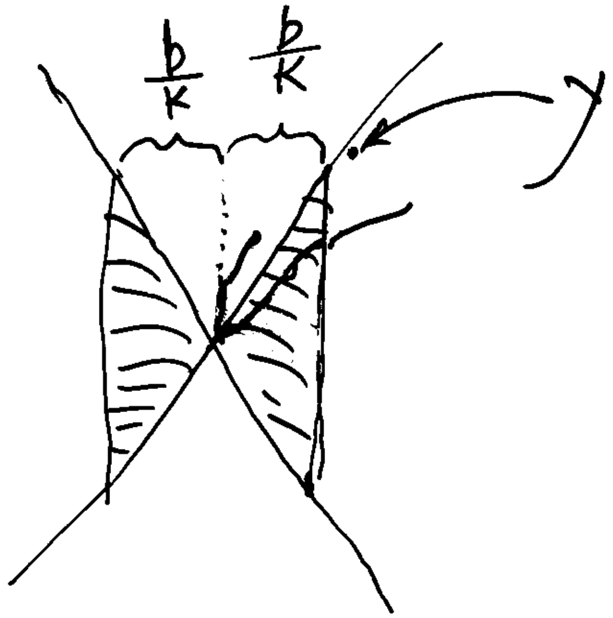
(1) (our ODE)

Implicit Function

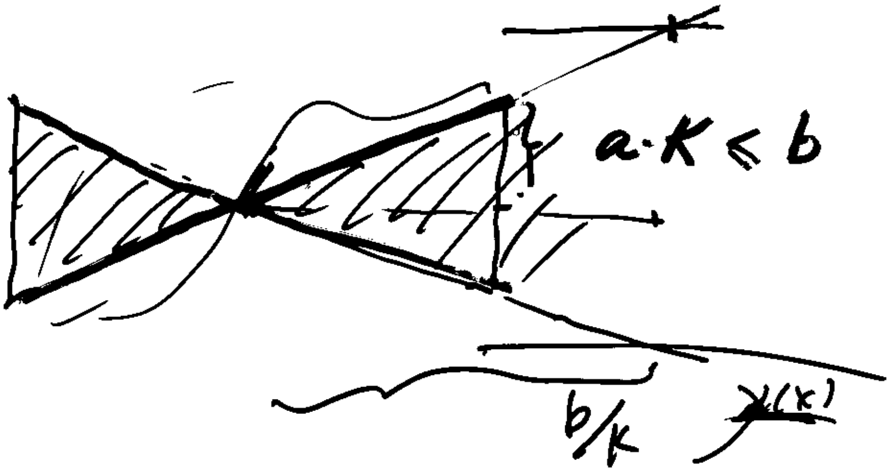
$$y = g(x)$$

$$a(x)y^2 + b(x)y + c(x) = \phi$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



if $f(x,y)$ is continuous
$\forall\ (x,y) \in R$
and
$y(x_0) = y_0$



and.

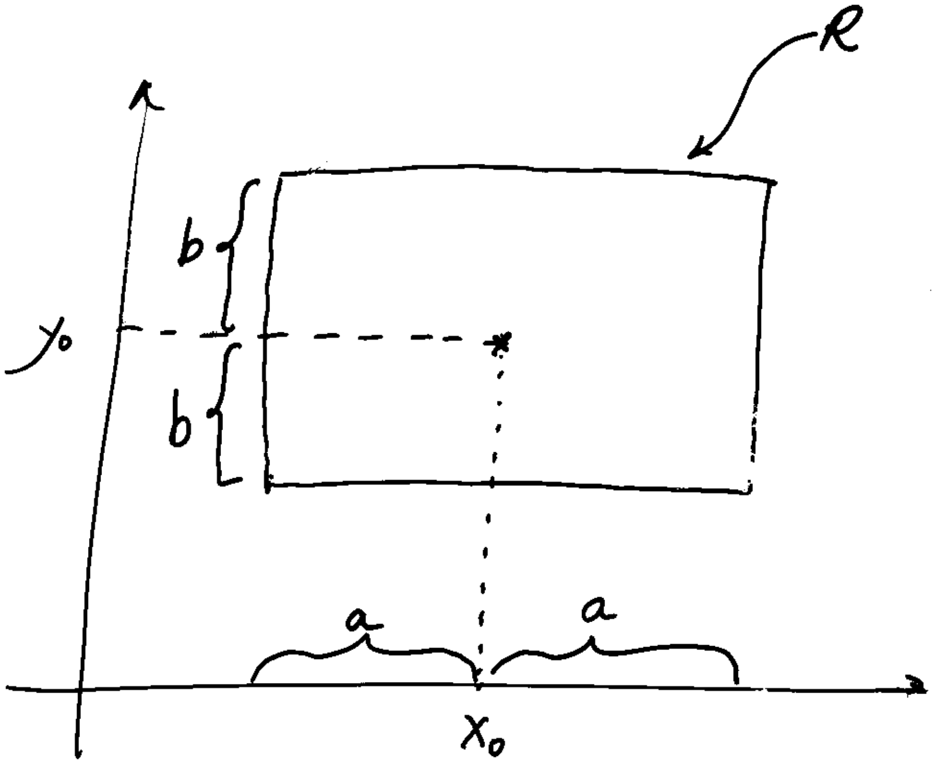
if $f(x,y)$ is continuous $\forall (x,y) \in R$

and

$y(x_0) = y_0$

$=$

$y' \leq K$



$y' = f(x,y)$ $\qquad$ $\underset{y'}{\underset{\shortparallel}{|f(x,y)|}} \leq K \quad \forall (x,y) \in R$

$$\frac{dy(x)}{d(x)} = y'(x) = f(x, y(x))$$

$$y(x) = \int dy = \int f[x, y(x)]\,dx$$

$$y(x) = y_0$$

$$y(x) = y_0 + \int_{x_0}^{x} f(x, y_0)\,dx$$

$$y(x) = y_0 + \int_{x_0}^{x} f(x, \underbrace{y(x)}_{\downarrow\ y_0})\,dx$$

$$F(x) = x$$

$$\underline{y = F(y)} \qquad F(x) = -x$$

$$y_1 = F(y_0)$$

$$y_2 = F(y_1)$$

$\cancel{y'' + ay' + by = \phi}$

$\underline{P(D)}[y] = \phi.$ $\quad P(D) = [D^2 + aD + b]$

if $\cancel{P(D)}$ $P(\lambda) = (\lambda - \lambda_1)^n Q(\lambda)$

$Q(\lambda_1) \neq \phi$

to show $\quad P(D)[xe^{\lambda_1 x}] = \phi$

$P(D)[\underline{xe^{\lambda x}}] \qquad \underline{x^{n-1} e^{\lambda_1 x}}$

$$= P(D)\left[\frac{\partial (e^{\lambda x})}{\partial \lambda}\right]$$

$$= \frac{\partial}{\partial \lambda}\left[\underline{P(D)(e^{\lambda x})}\right] \quad \begin{cases} \frac{\partial}{\partial y}\left(\frac{\partial f(x,y)}{\partial x}\right) \\ \frac{\partial}{\partial x}\left(\frac{\partial f(x,y)}{\partial y}\right) \end{cases}$$

$= \phi$ if $\lambda = \lambda_1$

$$P(D)[xe^{\lambda x}] = \frac{\partial}{\partial \lambda}[\underline{P(D)e^{\lambda x}}]$$

$$= \frac{\partial}{\partial \lambda}[\underline{P(\lambda)e^{\lambda x}}]$$

$\lambda = \lambda_1$

$$P(D)[xe^{\lambda_1 x}] = \phi$$