1), Computer virus problem gives below modelling equation:

$$\frac{dy}{dt} = k(N-y)y$$

$$\int \frac{dy}{(N-y)y} = k\int dt$$

$$-\int \frac{dy}{(y-\frac{N}{2})^2-(\frac{N}{2})^2} = k\int dt$$

By using the result ($\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right|$),

$$-\frac{1}{N}\ln\left|\frac{y-N}{N}\right| = kt + C$$

$$\ln\left|1-\frac{N}{y}\right| = -Nkt + C$$

$$\frac{N}{y} = -C_2 e^{-Nkt} + 1 \quad (C_2 = e^c)$$

$$y = \frac{N}{1-C_2 e^{-Nkt}}$$

Since $y(0) = N_0 \Rightarrow C_2 = 1 - \frac{N}{N_0}$

Hence, $$Y = \frac{N_0 N}{N_0 + (N-N_0)e^{-Nkt}}$$

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2), $2\sin(y^2)\,dx + xy\cos(y^2)\,dy = 0$

I.F. $= x^3$, so there exist $u(x,y)$ s.t. $\frac{\partial u}{\partial x} = 2x^3\sin(y^2)$

$$u = \int 2x^3\sin(y^2)\,dx$$

$$= \frac{x^4}{2}\sin(y^2) + k(y)$$

$$x^4 y\cos(y^2) = \frac{\partial u}{\partial y} = \frac{x^4}{2}(2y)\cos(y^2) + k'(y)$$

$$= x^4 y\cos(y^2) + k'(y)$$

$$\therefore k'(y) = k(y) = 0$$

Hence $u = \frac{x^4}{2}\sin(y^2) = C$

3), $(3xy+2y^2)dx + (4x^2+5xy)dy = 0$

I.F. $= x^2y^3$, so there exist $u(x,y)$ s.t.

$$\frac{\partial u}{\partial x} = x^2y^3(3xy+2y^2) = 3x^3y^4 + 2x^2y^5$$

$$u = \int 3x^3y^4 + 2x^2y^5\,dx$$

$$= \frac{3}{4}x^4y^4 + \frac{2}{3}x^3y^5 + k(y)$$

$$4x^4y^3 + 5x^3y^4 = \frac{\partial u}{\partial y} = 3x^4y^3 + \frac{10}{3}x^3y^4 + k'(y)$$

so $k'(y) = x^4y^3 + \frac{5}{3}x^3y^4$

$$k(y) = \frac{x^4y^4}{4} + \frac{x^3y^5}{3}$$

So $u = x^4y^4 + x^3y^5 = C$

(1.3) Q4, $\frac{dy}{dx} = (y+9x)^2$

Let $v = y+9x$, $v' = y'+9$

$$v'-9 = y'$$

So $v'-9 = v^2$

$$\int \frac{dv}{v^2+9} = \int dx$$

$$\frac{1}{3}\tan^{-1}\frac{v}{3} = x + C$$

$$y+9x = 3\tan(3x+C)$$

$$y = 3\tan(3x+C) - 9x$$

1.3), Q6), $y' = \frac{(4x^2+y^2)}{xy} = \frac{4x}{y} + \frac{y}{x}$

Let $u = \frac{y}{x}$, $y' = u'x + u$

$\therefore u'x + u = \frac{4}{u} + u$

$u'u = \frac{4}{x}$

$\int u\,du = \int \frac{4\,dx}{x}$

$\frac{u^2}{2} = 4\ln x + C$

$y^2 = 8x^2\ln x + Cx^2$

Q15), $e^{2x}y' = 2(x+2)y^3$, $y(0) = \frac{1}{\sqrt{5}} \approx 0.45$

$\frac{y'}{y^3} = 2e^{-2x}(x+2)$

$\int \frac{dy}{y^3} = \int 2e^{-2x}(x+2)\,dx$

$-\frac{1}{2y^2}$ $\cancel{\ }$ $= 2\int e^{-2x}x\,dx + 4\int e^{-2x}dx$

$= \int x\,d(-e^{-2x}) + 4\int e^{-2x}dx$

$= -xe^{-2x} + \int e^{-2x}dx + 4\int e^{-2x}dx$

$= -xe^{-2x} + 5\int e^{-2x}dx$

$= -xe^{-2x} - \frac{5}{2}e^{-2x} + C$

$x=0$, $y = \frac{1}{\sqrt{5}}$,

$\therefore -\frac{5}{2} = -\frac{5}{2} + C \Rightarrow C = 0$

$\therefore -\frac{1}{2y^2} = -xe^{-2x} - \frac{5}{2}e^{-2x}$

$y^2 = \frac{e^{2x}}{2x+5}$

$y = \frac{e^x}{\sqrt{2x+5}}$

Set 1.4,

Q4, $(e^y - ye^x)dx + (xe^y - e^x)dy = 0$

Since $$\frac{\partial(e^y - ye^x)}{\partial y} = e^y - e^x$$

$$\frac{\partial(xe^y - e^x)}{\partial x} = e^y - e^x$$

So Exact solution $u(x,y)$ exist.

$$\frac{\partial u}{\partial x} = e^y - ye^x$$

$$u = xe^y - ye^x + k(y)$$

$$\frac{\partial u}{\partial y} = xe^y - e^x + k'(y) = xe^y - e^x$$

$$\therefore k'(y) = k(y) = 0$$

$$u = xe^y - ye^x = C$$

Q12, $(e^{x+y} - y)dx + (xe^{x+y} + 1)dy = 0$

$$\frac{\partial(e^{x+y} - y)}{\partial y} = e^{x+y} - 1$$

$$\frac{\partial(xe^{x+y} + 1)}{\partial x} = e^{x+y}(x+1)$$

So no exact solution.

Let I.F. $= F(x)$,

$$\frac{1}{F} \cdot \frac{dF}{dx} = \frac{1}{xe^{x+y}+1}(e^{x+y} - 1 - xe^{x+y} - e^{x+y})$$

$$= -1$$

$$\int \frac{dF}{F} = \int -1\, dx$$

$$\ln F = -x$$

$$F = e^{-x}$$

Hence there exists u s.t. $\frac{\partial u}{\partial x} = e^{-x}(e^{x+y} - y) = e^y - ye^{-x}$

$$u = xe^y + ye^{-x} + k(y)$$

$$\frac{\partial u}{\partial y} = xe^y + e^{-x} + k'(y) = xe^y + e^{-x}$$

$$\Rightarrow k(y) = 0$$

$$\therefore u = xe^y + ye^{-x} = C$$

(5), Q8), $y' + 2y = 4\cos 2x$, $y(\frac{\pi}{4}) = 2$

$I.F. = e^{\int 2dx} = e^{2x}$

$\therefore e^{2x}y' + 2y e^{2x} = 4e^{2x}\cos 2x$

$\frac{d(ye^{2x})}{dx} = 4\int e^{2x}\cos 2x\,dx$

$ye^{2x} = \frac{4e^{2x}}{2^2+2^2}(2\cos 2x + 2\sin 2x) + C$

$y = \cos 2x + \sin 2x + C$

$y(\frac{\pi}{4}) = 2 \Rightarrow C = 1$

$\therefore y = \cos 2x + \sin 2x + 1$

Q14, $y' + y\tan x = e^{-0.01x}\cos x$, $y(0) = 0$

$I.F. = e^{\int \tan x\,dx} = e^{-\ln\cos x} = \sec x$

$\therefore \frac{d(\sec x \cdot y)}{dx} = e^{-0.01x}\cos x \cdot \sec x$

$= e^{-0.01x}$

$y\sec x = \int e^{-0.01x}dx$

$= -100e^{-0.01x} + C$

$y(0) = 0 \Rightarrow C = 100$

$\therefore y\sec x = 100 - 100e^{-0.01x}$

$y = 100\cos x(1 - e^{-0.01x})$