ERG 2011A, Fall Semester 2006

Homework #2

Due on Oct 09 (Monday), 2006 ; No Late submission will be accepted

Every Student MUST include the following statement, together with his/her signature in the submitted homework.

"I understand the University guidelines on academic honesty as shown in the web page http://www.cuhk.edu.hk/policy/academichonesty/ and I agree to comply with the guidelines. I declare that I do not plagiarize or cheat in any way that violates the spirit of the guidelines. I understand the severity of disciplinary action should the guidelines be violated.

Signed (Student ______________________)   Date: ______________________

Questions:

1) A) Prove that, if r_1 is a double root of the characteristic equation $f(r) = 0$, then $xe^{r_1 x}$ is a solution of the corresponding homogeneous linear differential equation.

[Hint: if $f(r) = (r-r_1)^2(a_0 r^{n-2}+\ldots..)$, then $f(r_1) = 0$ and $f'(r_1)=0$. Now e^{rx} can be considered as a function of r and x and from the rule $\frac{\partial^2 u}{\partial r \partial x} = \frac{\partial^2 u}{\partial x \partial r}$ for such a function we conclude that

$$L[xe^{rx}] = L[\frac{\partial}{\partial r}e^{rx}] = \frac{\partial}{\partial r}L[e^{rx}] = \frac{\partial}{\partial r}[f(r)e^{rx}] = f'(r)e^{rx} + xf(r)e^{rx},$$

where L[] is the linear ODE. Now set $r = r_1$.]

B) Extend the result of part (A) to the case of a multiple root of multiplicity k.

2) Consider the following differential equation:

$(x^3 \sin x)y''' - (3x^2 \sin x + x^3 \cos x) y'' + (6x \sin x + 2x^2 \cos x) y' - (6\sin x + 2x \cos x)y = 0$.........(**)

a) Show that $y=x^n$, for some value of *n*, is one of the solutions of the above equation.

b) Based on the findings of part a), solve (**).

Textbook:

Set 2.1) Q22

Set 2.2) Q29, Q30

Set 2.5) Q4

Set 2.7) Q3, Q7, Q16, Q20

Set 2.10) Q6